To R- geometry of plants

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The article gives a definition of the organic form as an infinite-similar and self-similar structure, which is based on the increased unity of the whole and the part - holomereological symmetry. A model of plant forms as organic forms is proposed, which are based on spherocylinders – infinite-similar cylindrical volumes joined with hemispheres at their ends. The data on the golden wurf in the metric of plant stems are presented. The cirrus leaf model in MathCad is briefly described. The problem of the organic form, the form of living organisms, is a long-discussed topic in biology, in particular, in morphology. There is a lot of empirical material relating to the description of a huge variety of biological forms. However, until now, the laws of the organic form and its specificity, in comparison with the forms of inorganic bodies, continue to raise more questions than answers. It is clear that organic forms are special, they have their own laws and types of morphogenesis. But what is the essence of these features, what is the logic of their organization, all this is still largely unclear.

Key words: *R-analysis, R-geometry, organic form, infinite-similarity, self-similarity, holomerological symmetry, spherocylinder, golden wurf*

1. Introduction

The author develops a new approach to the interpretation of the essence of organic forms, suggesting that these forms are small spaces with its symmetry, topology and metric, featuring considerably from those of the environment. For the expression of such specificity, a new mathematical tool, so-called *R-analysis* (*relativistic analysis of quantity*) is used, which is based on the isomorphic mappings (R-functions) between the finite structures and their infinite prototypes. On this basis, a new interpretation of the basic constructions of mathematical analysis is proposed, in particular, the calculus of infinitesimals [1-5].

Applications of the ideas of R-analysis to geometry allow to develop a new direction, which may be conventionally designated as "R-geometry" [6]. It is based on the relation between infinite spaces and finite volumes isomorphic to them (R-spaces). It is assumed that the shape of living organisms is a multi-level R-space, compiled by protoforms, as repeated at different levels one typed structures [7].

In this paper, organic forms of plant organisms are chosen for modeling as simpler and more geometrized types of living forms. But it is assumed that the results obtained for plants can be further generalized for all organic forms.

2. Materials and methods

When modeling organic forms, it is proposed to use isomorphic mappings (R-functions) $R^{\pm1}_{V*}$ between finite 3D-volumes V* and their infinite prototypical spaces V, where $V^* = R^{-1}_{V*}(V)$, $V = R^{+1}_{V*}(V^*)$. In this case, for the final volume V*, a coordinate system can be determined that most economically and organically expresses the internal geometry of the volume V*. Such a coordinate system can be called a *natural* (*selected*) coordinate system for the R-space V*. In simpler cases, vector R-functions can be reduced to scalar ones. For example, if the 3D-sphere S* with radius M>0 is considered as the volume V*, then it is natural to consider the spherical coordinate system (r, φ, θ) in the space S to be the selected system for S*, assuming that $R^{-1}_{S^*}(r,\varphi,\theta) = \left(R^{-1}_{M(r)},\varphi,\theta\right) = (r^*,\varphi,\theta)$, where $R^{-1}_{M(r)}$ is a scalar R-function that isomorphically compresses the real axis R into the interval (-M,+ M).

For example:

$R^{+1}_{M}(x) = (2M/\pi)\ tg\ (\pi x/2M)$ is the direct R-function,

$R^{-1}_{M}(x) = (2M/\pi)\ arctg\ (\pi x/2M)$ is the inverse R-function.

Similarly, a cylindrical volume $C^* = R^{-1}_{C*}(C)$ with radius M and height 2H can be built, where C is the space with the cylindrical coordinate system (ρ, φ, z), and $R^{-1}_{C^*}(\rho,\varphi,z) = \left(R^{-1}_{m(\rho)},\varphi,R^{-1}_{H(z)}\right) = (\rho^*,\varphi,z^*)$.

To model plant forms we will further use a *spherocylinder* - a cylindrical R-space C*, to the upper and lower bases of which the upper $S^{+}*$ and lower $S^{-}*$ R-hemispheres are attached, and they obtained from the R-sphere S* by narrowing zenith angle θ to values θ∈[0,π/2] and θ∈[π/2,π] respectively (fig. 1).

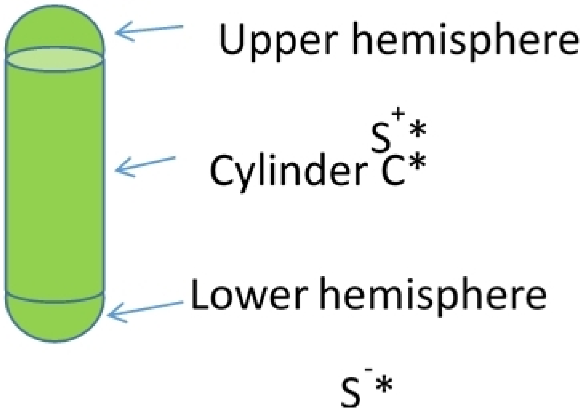

Fig. 1. Spherocylinder

3. Literature review

The specificity of the organic form was noted for a long time. For example, in the research of I.V. Goethe, a theory of metamorphosis was proposed, i.e. the transformation of one plant organ into another. Goethe also believed that the protoform of the plant is the leaf from which its other organs originate: calyx, corolla, stamens, and pestle. An attempt to express plant forms as formed on the basis of a single protoform is also vividly presented in the so-called *telome theory* of the German botanist V.Zimmermann [8], in accordance of which all the basic forms of the plant are formed on the basis of cylindrical structures: terminal (*telomes*) or intermediate (*mesomes*). V.I.Vernadsky noted the peculiarity of organic forms associated with the presence of their own symmetry and local geometry. We can not pass the studies of morphogenesis in living organisms by D'Arcy Thompson, presented in his famous work, "On Growth and Form" [9]. He puts forward the idea of regular transformations of some species forms into others, continuing the principles of metamorphosis on the supraorganismic level. The paper of S.V.Petukhov [10] introduced the concept of the *gold wurf* $W(a,b,c) = (a + b)(b + c)/b(a + b + c) = \varphi^2/2 \approx 1.309$, where a, b, c are the values of three adjacent divisions of the organic form, $\varphi \approx 1.618$ is the proportion of the golden ratio. The reproducibility of the value of the golden wurf for many biological divisions, as well as the importance of conformal and projective mappings in the transformations of the organic form, is noted in [10]. Numerous examples of biological symmetry, attempts of their generalizations are shown by N.A.Zarenkov, particularly in [11]. He develops the ideas of "biosymmetry" proposed by Yu.A.Urmantsev in his version of the general theory of systems (GTS). An indication of the advantages of the component approach when transferring a geometric shape from one medium to another is found, for example, in [12]. Interesting data on the coordination of parameters of organic and inorganic forms within the framework of temporal definitions can be found in [13-15]. You can also mention the research of A.A. Lyubishev, L.V. Belousov, dedicated to the specificity of the organic form. But in general, it should be noted that the field of biological morphology still remains at a predominantly descriptive level and urgently needs the first theoretical generalizations.

4. The phenomenon of organic form

The organic form can be defined as a spatial structure, which has the following essential properties: 1) *infinite-similarity*, i. e., in our case, this structure is a finite volume V*, which is isomorphic to the infinite space V, i. e. $V^* = R_{V^*}^{-1}(V^*)$, where $R_{V^*}^{-1}$ is an isomorphism, 2) *self-similarity*: in the form V*, there are parts v* that are similar to the whole V*, i.e. an isomorphic mapping $V^* = r_{V^*}^{-1}(V^*)$ is defined (note that self-similarity is a property of the whole V*), 3) *holo-similarity*, when a part of the whole is similar to the whole (holo-similarity is a property of part v*), 4) the presence of *protoform*, i.e. such a form v_0*, by transformations of which all the holo-similar parts v* of the whole V* (including the whole V * itself) are formed.

All the described features of the organic form express one general principle*: the increased interpenetration of the whole and the part* when the third state of the *whole-part* arises (or *holomerone*, from the greek "holos" - the whole, and "meros" - the part), i.e. an invariant of transformations between the whole and the part. This kind of invariance can also be called *holomereological symmetry*.

In addition, the organic form as a whole V* also has infinite-similarity, i.e. it is an infinite space compressed into a finite volume, representing additional kind of invariance, the invariance between infinite and finite. These kinds of invariants can be called *fin-infinites,* or even shorter: *finfinites*.

In this case, in the most concise way, it can be argued that the *organic form is a finfinite holomerone*.

5. Plant as an organic form

The abstract idea of an organic form, as defined above, can be more specifically implemented in the mathematical modeling of a generalized plant as a system of coordinated spherocylinders.

According to Zimmermann's telome theory, a plant is a system of coordinated cylinders. However, the cylindrical coordinate system cannot generate, as its own movements, a laterally deviation of one cylinder relative to another. For such a deviation, it is necessary that the z axis of the lateral cylinder deviates by a nonzero zenith angle relative to the z axis of the original cylinder, but there is no zenith angle in the cylindrical coordinate system, it is only in the spherical one. Therefore, *considering the plant form as an objectified own coordinate system*, we must supplement the cylindrical structures of the telome theory with spherical structures. The design of the spherocylinder described above, just allows you to implement such a deviation. Due to the presence of hemispheres at the ends of the cylinder, lateral deviations of the z axis are possible here with the formation of a new spherocylinder. Thus, we can assume that wherever plant structure is capable of branching, it can do so only within a spherical component (hemispheres) of spherocylinders (fig. 2).

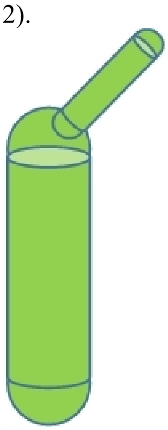

Fig. 2. Docking of central and lateral spherocylinders

The fact that a part of the stem can be cut out of the plant, planted in the ground, and new shoots will start to grow from its upper end, and the roots begin to grow below, suggests that inside some spherocylinders there are potentially others that can be activated under certain conditions. This is the property of self-similarity of the plant form as organic form.

Morphologically, the lateral branchings of the plant stem realize themselves through the formation of leaves and the growth of the new shoot from the lateral bud, which is located in the sinus between the leaf stalk and the stem. The leaf itself expresses the finale of plant growth, while the lateral bud contains the germ of a new shoot with its growth potential. But in order to form a new shoot in a new direction of growth, you need a leaf and a lateral bud that appears near it. Like a leaf is a direction with a fixed value, the lateral bud and its future shoot are a value with a fixed direction.

It seems that it is very convenient to simulate by changing of the vector X, where |X| is the value of X, and x is the unit vector of X, i.e. x = X/|X|. Then we get:

$$\frac{dX}{dt} = \frac{dx|X|}{dt} = |X|\frac{dx}{dt} + \frac{xd|X|}{dt}.$$

Here, the complete change of the vector in time is the sum of two changes: 1) change in direction with the fixed value, which is expressed by the term $|X|\frac{dx}{dt}$, 2) change in the value with the fixed direction $\frac{xd|X|}{dt}$.

So in the dynamics of plant growth, these two components are morphologically separated. The moment of direction change with the fixed value is allotted to the leaf with the formation of a lateral bud, while the growth of the lateral shoot from this bud expresses the moment of change in the value with the fixed direction.

As you know, the stem has *negative geotropism* (the tendency to grow up, against gravity), and the root has *positive* one (grow down, by gravity). This means that the stem and root carry their own coordinate systems, where there is a vertical axis, and they coordinate this axis with the direction of gravity. In the cylindrical coordinate system, there is such an axis, this is the z axis. But we need also the asymmetry of the axis z to express geotropism, what can be done through the introduction of two cylindrical coordinate system: (ρ, φ, z_L) for stem spherocylinder, and (ρ, φ, z_G) for the root one, where $z_L = -z_G$.

But if there are essentially two oppositely polarized cylinders with coordinates z_L and z_G in the cylinder of the spherocylinder, then each of them is associated with only one hemisphere (upper for itself). As a result, we have two *hemispherocylinders* in one *spherocylinder*. They can be called, for example, as *L* - and *G-hemispherocylinders* (*levitational* and *gravitational* ones).

Another interesting phenomenon of plant morphogenesis is *phyllotaxis*, i.e. patterns of disposition of leaves on the stem. It turns out that in the general case, the attachment points of leaf stalks to the stem form a spiral, where m turns of spiral have n leaves, if we go along the stem (as z axis) from a leaf with a certain azimuth angle φ in the cylindrical coordinate system (ρ, φ, z) to the first leaf with the same angle. They say about such leaves that they lie on one vertical line - *orthostichia*. It is remarkable that pairs of numbers m and n (usually they are written as the fraction m/n) in this case are elements of the Fibonacci series and are characteristic of each type of plant. For example, the leaf disposition in cereals, birch, grapes is expressed by the formula ½, in a tulip, alder - 1/3, in pears, currants, plums - 2/5, in cabbage, radishes, flax - 3/8, etc.

In this case, we see the involvement of the azimuthal angle in the organization of spherocylinders. On the stem of the plant upwards, there is a spiral organization, expressed in lateral leaves (shoots). It should be noted that a cylindrical spiral is a fairly organic structure in the cylindrical coordinate system. It is expressed by the form (ρ_0, φ, z(φ)), where for example z(φ) = kφ.

Let us see how a lateral shoot is formed. In the area of the stem node, there is a cross section of the cylinder, which becomes the base of the upper hemisphere, due to which the z axis can deviate, and a lateral spherocylinder appears. This creates an uncompensated spherical polarity (in the plane (ρ, φ)), which must be compensated, and the emergence of a sequence of other side leaves (shoots) is the intention to gradually compensate for the initial polarity until the branch will come exactly above the initial one (on one orthostich). Similar relationships can be expressed by *polar analysis* [3].

Summing up, we can assume the plant structure as the organic form having increased unity of the whole and parts, as well as infinite-similarity whole, more specifically expressed in the possibility of providing R-spherocylinders as a base plant protoform, by various modifications and compositions of which derivative plant forms are generated. In this case, the plant appears as a multi-level hierarchical R-space, which includes many spherocylindrical R-subspaces and their compositions. Each of the vertical R-spherocylinders includes two hemicylinders with opposite geotropy and their own cylindrical coordinate systems that are oppositely directed along the z axis.

6. Golden wurf in the proportion of plants

S.V. Petukhov introduced the concept of the *golden wurf* and showed its implementation for many divisions of organic forms [10]. The author investigated the implementation of the golden wurf W(a, b, c) for neighboring divisions a, b, c using the example of the lengths of internodes of plant stems and obtained a good agreement between the average values of the wurfs and 1.3. Below is one of the tables for the chicory stalk (table 1). After the table, there is a graph representing the values of the Wurfs from this table (fig. 3).

Table 1. The lengths of neighboring internodes x_k and the wurfs of their triples for the chicory stem

k	Length x_k of neighboring internodes, cm	Wurf W(x_k,x_{k+1},x_{k+2})	Wurf average
1	4.8	1.240	1.331
2	6.7	1.407	
3	5.8	1.270	
4	6.8	1.430	
5	5.8	1.250	
6	7.3	1.430	
7	6	1.290	
8	7.3	1.270	
9	7.3	1.390	
10	5.5	1.340	

k	Length x_k of neighboring internodes, cm	Wurf $W(x_k,x_{k+1},x_{k+2})$	Wurf average
11	5.4	1.280	
12	5.4	1.380	
13	4.2	1.340	
14	4	1.270	
15	4	1.450	
16	2.9	1.270	
17	3.4	1.250	
18	2.9	1.570	
19	1.7	1.200	
20	2.3	1.240	
21	1.5	1.400	
22	0.7		
	0.5		

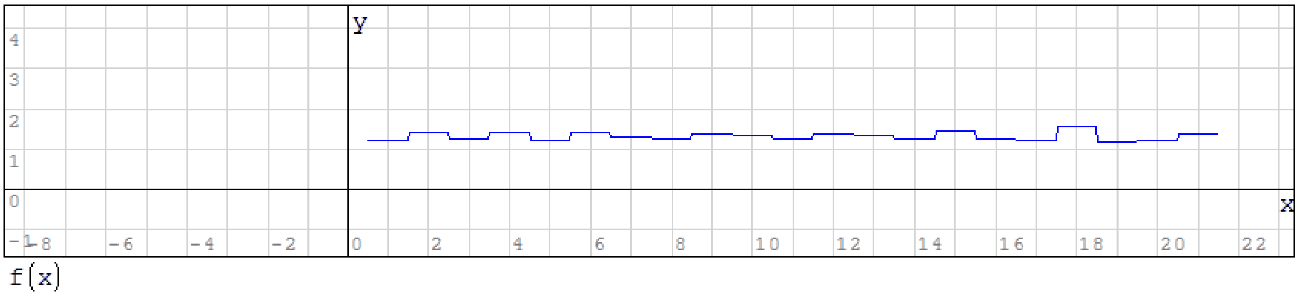

Fig. 3. Graphical representation of the data from table 1

7. Cirrus leaf modeling

In a number of my works [6, 7, 16], I described leaf models with arc-like venation as a result of the action of the inverse R-function on the 2D-plane, justifying the infinite-similariy of the leaf as a special case of the organic form. Below is a brief description of the simulation in program MathCad of more complicated case of cirrus leaf, which can not be received by the effect of one R-mapping, and it has to connect this type of transformation of infinite structures to finite ones with external transformations of the finite structures, generally yielding a mixed strategy of organic forming.

At the beginning, when constructing a mathematical model of cirrus leaf, it is carried out the compression by inverse R-function $R^{-1}{}_M$ of the axis x, yielding the variable $x^* = R^{-1}{}_M(x)$. Then, the contour of the sheet $K(x^*)$ is formed, taking, for example, a linear function, raising it to a power β of less than 1, which will give the convexity of the contour:

$$K(x) := M0\cdot\left(\frac{M - R(x)}{M}\right)^{\beta}.$$

Here M0 is the maximum level of the sheet edge along the y axis, R(x) is the inverse R-function $R^{-1}{}_M(x)$.

In MathCad, we obtain the graph $K(x^*)$, expressing K(x) along the axis y, and along the axis x we have value $x^* = R(x) = R^{-1}{}_M(x)$. Here x acts as a parameter, on which both the absciss and the ordinate depend. But the function K is written for x as its argument. And to get a point of the contour $K(x^*)$, we need to set the value of the argument x for the function K. It should be borne in mind that the functions K(x) and $K(x^*)$ are different!

$$K(x^*) = M_0\left(\frac{M - x^*}{M}\right)^{\beta}.$$

Knowing the contour K (x^*) and the point on the central vein x^*_i, we can compress along the y axis, above this point, a segment of length from x^*_i to $K(x^*_i)$ by the inverse R-function $R^{-1}{}_{K\,(x^*i)}$, for which it is necessary to use again the parametric definition of functions, for example, from the same x.

Here, it will be necessary to determine the function along the y axis with respect to some argument Xi(x), which will give the value x^*_i for all x.

Accurate hit on the contour, i.e. at the value of $K(x^*_i)$, is achieved in this case for the function $R^{-1}{}_{K(S(x^*i))}(x) = 1/R_{K(xi)}(x)$, and not for $R^{-1}{}_{K(x^*i)}(x)$, where $S = R^{+1}{}_M$ is the direct R-function.

So, in MathCad, we are building the function

$$Ri(x) := \left(\frac{2\cdot K(S(xi))}{\pi}\right)\cdot atan\left(\pi\cdot\frac{x}{2\cdot K(S(xi))}\right),$$

$$X0(x) := xi,$$

where xi is understood as the value on the R-compressed axis, which will correspond to S(xi) for the parameter x. Therefore, we write not K(xi), but K(S(xi)).

Therefore we get the vertical segment over xi up to K(xi). This segment expresses the orthogonal lateral vein at the point of separation xi from the central vein.

To express cirrus venation, we tilt this segment from point xi to point x(i +1). To do this, we introduce a new parameter ai, for which we set the range of variation and a linear function on this range:

$$ai := S(xi), S(xi) + 0.1 .. S(xi + 1),$$

$$Zi(ai) := \left[\frac{R(ai) - xi}{(xi + 1) - xi}\right]\cdot K(S(xi + 1)).$$

Here again, it should be noted that we take the parameter ai relative to the R-compressed scale x*, defining it in the interval from $(S(x_i))^* = x_i$ to $(S(x_{i+1}))^* = x_{i+1}$, therefore, the parameter ai itself must be determined with respect to the direct R-maps for this segment.

We also supplement all the constructions for y≥0 on the region y<0 by putting a minus sign in front of the corresponding functions. As xi, points 0, 2 and 4 were selected. The last inclined straight is ended at the point K(5.5) of the edge. As a result, we have the following picture for M = 6 and M_0 = 5 – fig. 4.

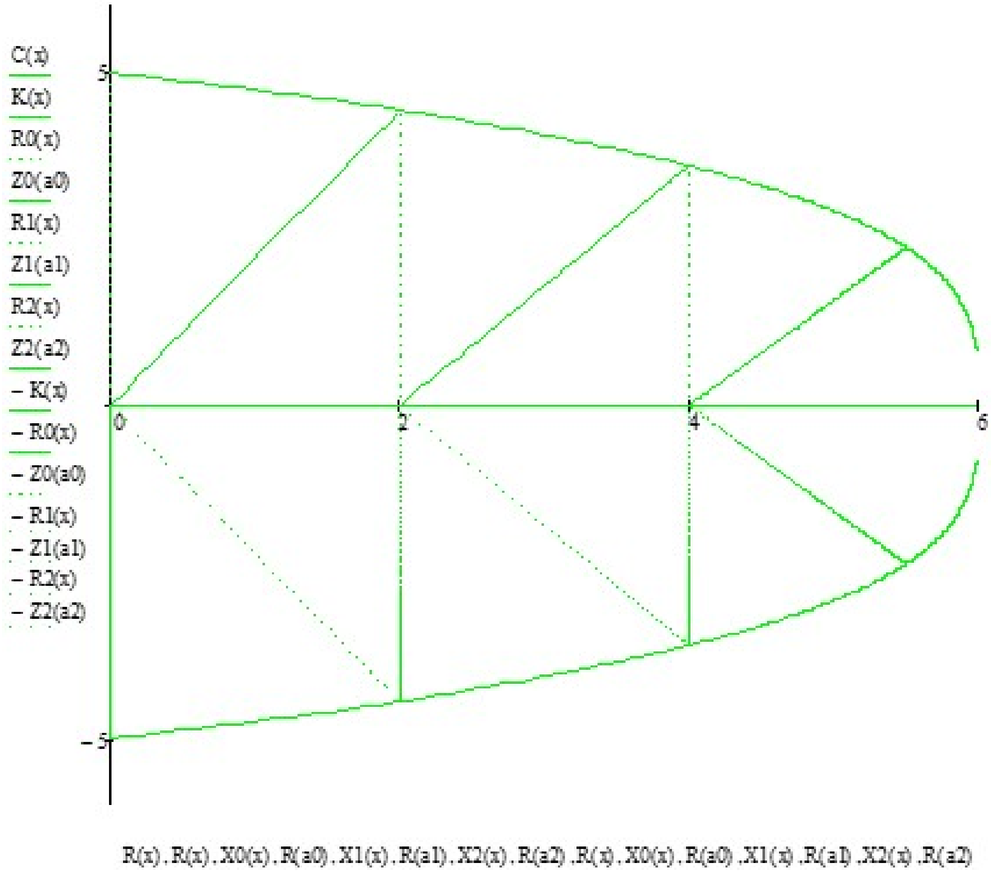

Fig. 4. Cirrus leaf model

Summing up, we can conclude that the laws of the plant form, as an important example of the living form in general, completely correspond to the hypothesis of the organic form and infinite-similarity of biological systems, striving in the limit to maximize fusion of the whole and the part in the state of holomereological symmetry, combining the properties of infinite-similarity and self-similarity, reinforced by penetration of a single spherocylindrical protoform. The plant appears as a kind of "living crystal", possessing an amazing "living geometry" with its own peculiar laws and principles of a highly holistic being.

8. Results

The article defines the organic form and its application to modeling the morphology of a generalized plant as an infinite-similar and self-similar structure on R-spherocylinders. On this basis, an attempt was made to explain a number of plant morphology phenomena: lateral bud, geotropism, phyllotaxis, etc. Empirical data are presented, supporting the golden wurf phenomenon for plant morphology. A model of the cirrus leaf as a system of R-structures is constructed.

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