Visualizing methods of multi-criteria alternatives for pairwise comparison procedure

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The article deals with the problem of choosing a preferred alternative in a pairwise comparison procedure. The difficulties of applying this procedure in a case of using alternatives with a large number of criteria are noted. It is proposed to supplement the procedure of expert pairwise comparison with visualization tools of multi-criteria alternatives. The paper considers several visualization methods for multi-criteria alternatives for pairwise comparison procedures: histograms, two-dimensional graphs, three-dimensional surfaces, probability distribution diagrams, visualization based on modifications of radar and radial diagrams, as well as combined methods. It described an experimental study of the application of the considered method for the task of determining the preferred alternative by the example of choosing one of two OpenFoam solvers (rhoCentralFoam and pisoCentralFoam), with the help of which estimates of the accuracy of calculating the inviscid flow around a cone were obtained. Each solver is characterized by 288 criteria. It is shown that the use of some of the methods considered does not make it possible for the expert to make a choice. In this case, a good result was obtained using methods for constructing three-dimensional surfaces, probability distribution diagrams, as well as using the combined method based on modified radar diagrams. It is concluded that the rhoCentralFoam solver is more preferable if there are no additional criteria for ranking the criteria. The possibility of using the combined method in combination with the ranking procedure of criteria (or their groups) during decision-making is also noted.

Keywords: *multi-criteria alternatives, visual pair comparison, alternatives visualization, decision support.*

1. Introduction

In decision theory, one of the basic tasks is ranking alternatives [1, 2]. This allows setting their priority in relation to the task at hand. There are various methods that allow such ranking, some of which are expert. Among the expert ranking methods, the method of pairwise comparisons [3] has proven itself well, the essence of which is to provide the expert alternately with pairs of alternatives for comparison, during which he prefers one of them. In particular, this procedure used in one of the classical decision-making methods — the hierarchy analysis method developed by T. Saati [4], in which it is necessary to construct matrices of pairwise comparisons for all levels of the hierarchy.

The use of pairwise comparisons is usually effective in cases where each alternative is well reflected in the expert's perception or is characterized by a small number of criteria (usually no more than 10) [5]. In the case when an expert needs to compare new for him alternatives with a large number of criteria, this can cause difficulties for him. Therefore, in such situations, it is necessary to use additional tools, for example, reducing the dimension, or statistical processing of criteria values. However, even applying these approaches, there is still a chance of not getting the desired result. For example, in the case of calculating statistical characteristics, we can get conflicting data in a situation where the mathematical expectation for an alternative is better, but the variance is worse. Therefore, additional tools are needed that could help the expert decide.

One such methods may be visual analytics - when for each alternative a corresponding visual image is constructed that characterizes the set of values of its criteria. Visualization is able to present the alternative as a holistic image, and it will be easier for an expert to make his choice with its help. It should be noted that the visual comparison of alternatives is currently already being applied and shows a good result, for example, when comparing the site design at the stage of its design [6].

Thus, we have the task of visualizing data sets characterizing alternatives. Consider and analyze several approaches and methods that can be applied to visualize multi-criteria alternatives.

2. Visualization methods

Data preparation

The visualization procedure begins with a step requiring initial data preparation.

1. All input values should be given to a numeric format - relevant methods of decision theory may be used for these purposes [7].
2. Normalization of data per segment [0; 1] taking into account the direction of the criterions optimization (maximization or minimization). For these purposes, the formulas can be used:
   - $v'_{i,j} = \frac{v_{i,j} - v_{min,j}}{v_{max,j} - v_{min,j}}$ – in case of maximization;
   - $v'_{i,j} = \frac{v_{max,j} - v_{i,j}}{v_{max,j} - v_{min,j}}$ – in case of minimization,

where i – alternative number ($1 \le i \le N$), j – criteria number ($1 \le j \le K$), N – count of alternatives, K – count of criteria, $v_{max,j}$, $v_{min,j}$ – maximum and minimum possible value of j-th criteria. Values $v_{max,j}$, $v_{min,j}$ usually determined on the basis of their physical meaning. However, if there are problems with their definition, then they can be calculated by the formulas:

$$v_{min,j} = \min(v_{i,j}),$$

$$v_{max,j} = \max(v_{i,j}).$$

After the data is prepared, you can proceed to visualize them. For these purposes, several different approaches and methods can be applied, in each of which we will consider the visualization of two alternatives and the features of their visual pairwise comparison.

Histograms

One of the most accessible and simple methods of visualization of multidimensional data is diagrams, and among them, the most accessible in the procedure of pairwise comparison can be called histograms. In this case, two main approaches to their application can be distinguished.

1. Both alternatives are shown on a common histogram. (Fig. 1). In addition, you can use the option of overlapping columns to focus on the deviation of values by criteria.

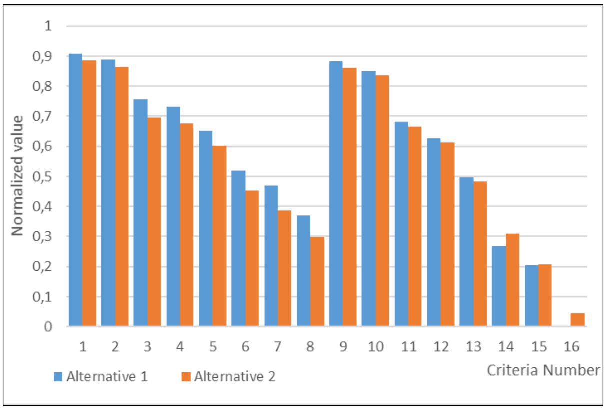

Fig. 1. Visual comparison of two alternatives in a common histogram

2. Each alternative is visualized on separate histograms. (Fig. 2).

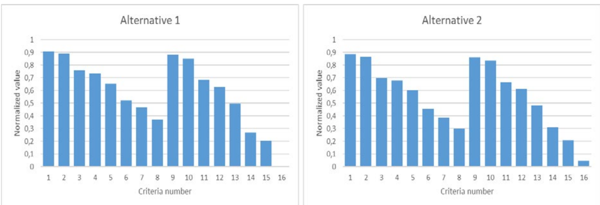

Fig. 2. Visual comparison of two alternatives on separate histograms

Note that in the 2nd approach, it is important to use identical parameters of the diagrams (color, size, etc.) in order to reduce the subjectivity of the perception of visual images, describing the data sets of the corresponding alternatives, and to enable the expert to focus on a holistic perception of visual images.

When determining preferences among alternatives based on visual images, an expert can be guided by his perception of the degree of filling of the columns of diagrams, including and the total area of all columns (the second approach is more suitable for these purposes) or the subjectively averaged deviation of the criteria columns of the two alternatives (the first approach is more suitable for these purposes).

The use of histograms is justified for those situations when it is necessary to see the whole alternative as a whole, and the number of criteria is not too large (about 5-20). Moreover, the effectiveness of this visualization method is additionally achieved if ranking approaches or non-linear scales were used in the normalization process since this allows you to get quite noticeable differences in the criteria that the expert can undoubtedly notice with a pairwise comparison.

In addition, when comparing two histograms, the expert's perception can be significantly affected by the order of columns (criteria), therefore, when using this approach, it is appropriate to think over the ordering or grouping of criteria based on their features, for example, by a degree of importance.

In addition to using this visualization method in the pairing comparison procedure, it can also be useful in ranking the criteria, as well as grouping them. This makes it possible to use this method as a preparatory stage for the construction and comparison of visual images of alternatives.

Two-dimensional graphics and three-dimensional surfaces

Other common data visualization methods are two-dimensional graphs and three-dimensional surfaces. Their application can be effective when there are areas with a noticeable difference in the values of the criteria, and due to interpolation, these areas are more pronounced.

In the case of two-dimensional graphs, the X axis can be interpreted as the serial number of the comparison criterion j $(1 \leq j \leq K)$ and at Y axis normalized criterion value – $v'_{i,j}$ $(1 \leq i \leq 2)$ is plotted. If each criterion corresponds to a numerical value (for example, time), then it can also be used to determine the X coordinate (only provided that all these values are pairwise different) (Fig. 3).

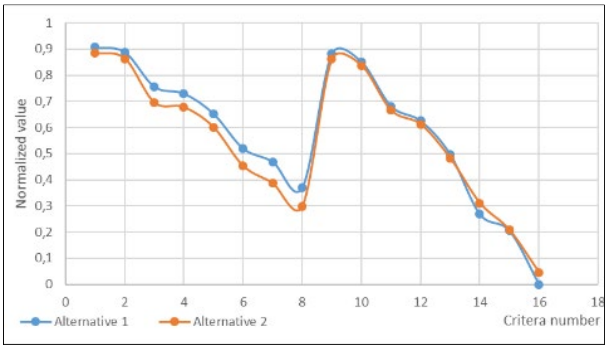

Fig. 3. Visual comparison of two alternatives on a two-dimensional graph

However, for the application of visualization based on surfaces in three-dimensional space, a prerequisite is the presence or ability to display a set of criteria on the axis X and Y: $fx(j) = x,\ fy(j) = y,\ 1 \leq j \leq K$. This mapping can be set based on the physical meaning of the criteria, or by using the grouping of criteria (for example, using clustering methods) (Fig. 4).

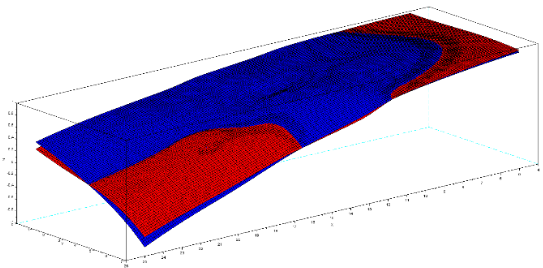

Fig. 4. Visual comparison of two alternatives using a surface in three-dimensional space

Similar to histograms, visual images for the pairwise comparison procedure in the case of two-dimensional graphs and three-dimensional surfaces can also be constructed both on one diagram, and on two neighboring ones with the same parameters.

The expert can make his choice based on the area (for surfaces) or the intervals (for curves) of the zones where one alternative prevails over another. Moreover, in the case of surfaces built in three-dimensional space, a prerequisite is the availability of tools that allow you to rotate and zoom in on the surface so that the expert can choose the most suitable angles for comparison. Thus, it is important to provide an interactivity property. This method allows you to compare visual alternatives entirely based on the subjective perception of the coverage area of one surface of another, and also with its help you can determine the various relationships of the groups of criteria that are used as the coordinates of the measurements along the abscissa and ordinates.

Probability distribution diagram

Probability distribution diagrams can also be used as another approach to visualizing data sets describing alternatives. For these purposes, the interval [0; 1], to which all normalized values of the criteria ($v'_{i,j}$) belong, on M equal intervals in length (for example, by 5 or 10) depending on the number of criteria. These intervals are located on the abscissa axis. After that, the number $C_{i,m}$ ($1 \leq m \leq M$) of hits of the normalized criteria values for each of the intervals is determined, which then allows you to determine the corresponding probabilities: $p_{i,m} = \frac{C_{i,m}}{K}$, used as values on the ordinate axis when plotting (Fig. 5).

As in previous approaches for this visualization method, it is also possible to build diagrams on a single diagram, or separately. The choice of preference in the comparison of the probability distribution the expert can give an alternative, which is characterized by the displacement of the probability distribution to the right (towards the interval with the highest criteria of values). This type of chart is conveniently displayed on a single diagram while applying transparency of the columns so that the differences are accented (Fig. 5).

The effectiveness of this approach becomes significantly higher when the number of criteria K is sufficiently large (for example, hundreds and thousands). With its help, not only visual images can be obtained, but also quantitative probabilistic characteristics of the dominance of one alternative over another. Also, this method allows us to group criteria, but it does not allow them to be ranked.

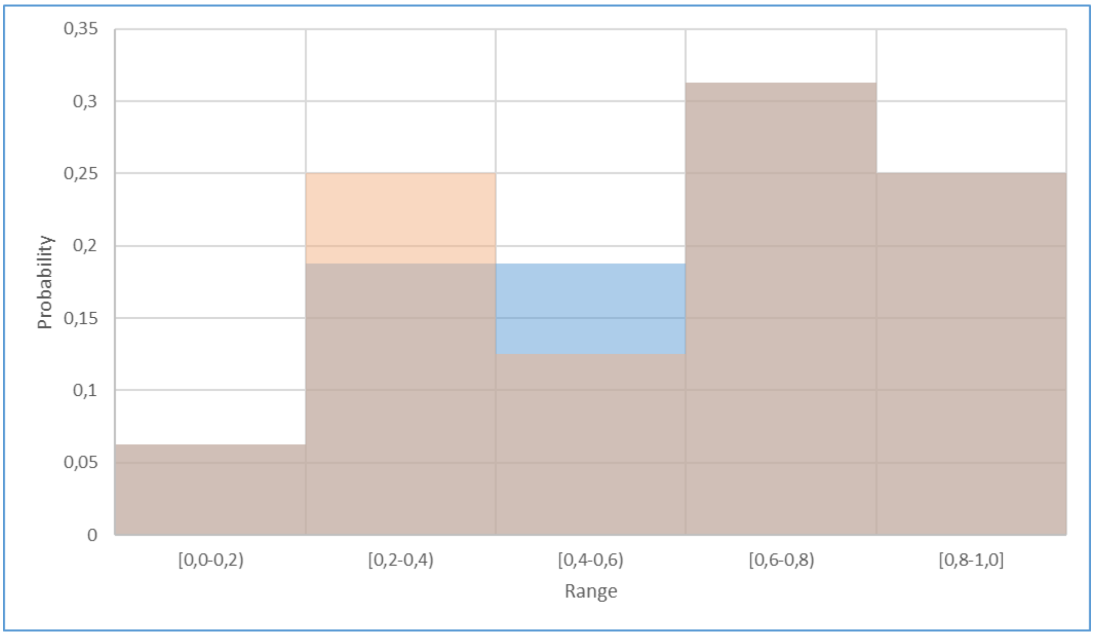

Fig. 5. Visual comparison of two alternatives using a probability distribution diagram

Visual images based on radar and radial diagrams

As noted in [8] visual images of alternatives can also be built on the basis of methods based on the radar and radial diagrams. The following visualization options are proposed:

- sectors with radii proportional to the criteria of alternative (Fig. 6);
- sectors with radii proportional to the roots of the criteria of alternative (Fig. 7);
- radar diagram with a permutation of criteria by grouping large values side by side (Fig. 8);

- radar diagram with a permutation of criteria, taking into account their alternation (alternately clockwise are the criteria with larger and smaller values) (Fig. 9).

When using this method of visualization, the main emphasis is on the fact that the best alternative occupies a larger area, and also that the visual image is brighter due to the use of gradient fills (in the center, the color is more neutral – green, and on the periphery – more contrast – red).

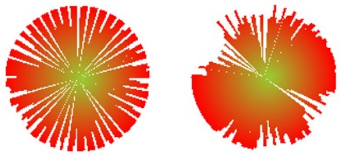

Fig. 6. Visual comparison of two alternatives using a radial diagram with sector radii proportional to the values of the criteria

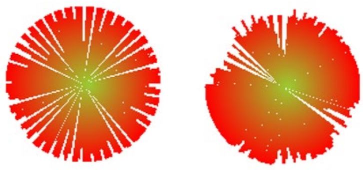

Fig. 7. Visual comparison of two alternatives using a radial diagram with sector radii proportional roots of values of the criteria

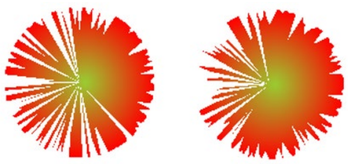

Fig. 8. Visual comparison of two alternatives using a radar diagram with a permutation of criteria by grouping large values side by side

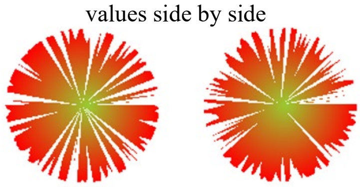

Fig. 9. Visual comparison of two alternatives using a radar diagram with a permutation of criteria, taking into account their alternation

If one of these visualization methods is used, the expert, when paired, selects the alternative that seems to him subjectively brighter and larger in area.

The methods described in [8] represent a more universal visualization mechanism, because they allow one to take into account the order and grouping of criteria, and also for them integral quantitative characteristics (brightness, area) can be determined.

Complex method

When comparing alternatives by only one visual image, it is not always possible to choose the preferred one from them. This is because the comparison is usually based on the color, shape, area or volume of the visual objects defining the respective alternatives. At the same time, different visualization methods have different advantages and disadvantages, and often some of them may not be useful in the visual comparison itself, but in the preparatory stage, the purpose of which is to determine the order or grouping of criteria (as histograms and 3D surface), as well as the integral quantitative characteristics of visual images - brightness, area of prevalence, statistical characteristics (probability distribution diagram), etc. And already these characteristics allow, for example, to set a specific order of permutation and grouping of criteria during visualization (radial and radar diagrams).

Thus, it is advisable to move from the task of comparing a single visual object to the task of comparing a group of visual objects that characterize an alternative or its components. For this purpose, an integrated approach is proposed, consisting in the sequential presentation of a series of visual images obtained using different methods, until the expert makes a choice.

For this expert to select the first represented whole visual images. If with their help he cannot determine the preferred alternative, then by means of visual analysis he tries to identify groups of general criteria, and also, if possible, to rank and filter them. Further, the selected groups can be visualized separately and placed in a table grid - on the right are the images for the components of one alternative, and on the left for the other (Fig. 10).

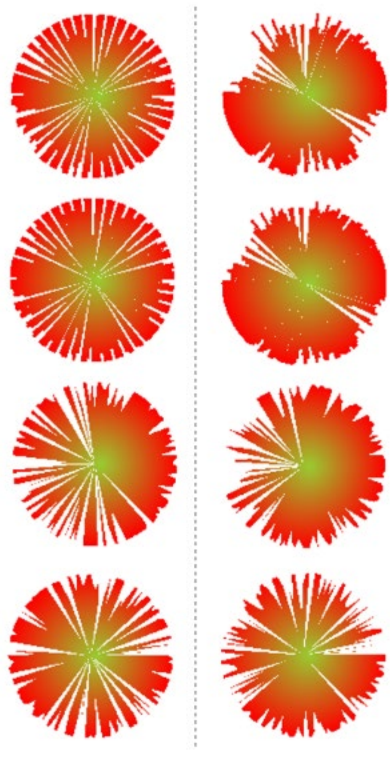

Fig. 10. Visual comparison of two alternatives using the integrated method

In such a set of visual images, there is a high probability that in a number of rows it will be possible to choose a preference. If for one alternative there are more such preferences than for another, then you can make a choice in favor of this alternative.

3. Experiments

Let us analyze the application of the considered methods on the example of the alternative (solvers) described in the works [9, 10]. As noted in [8], out of five solvers, two give the best results – rhoCentralFoam and pisoCentralFoam (rCF и pCF). Given the fact that the number of comparison criteria for these two alternatives is quite large, we will use visual images built on different diagrams. In Fig. 11 is a visual comparison using histograms.

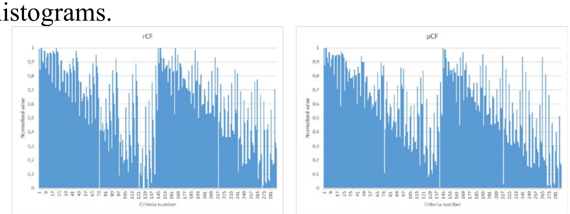

Fig. 11. Visual comparison of two solvers using histograms

For most experts, this comparison will not be unambiguous, because the images are very similar, and at the same time on both diagrams, there are both areas with the best values and the worst.

We will get an approximately similar result when using a two-dimensional graph, however, constructing a surface in three-dimensional space can give a more interesting result. This method visualization is possible because criteria can be grouped due to the fact that they were obtained during computational experiments by varying two parameters – angle β (in range 10-35° with step 5°) and Mach numbers (in range 2-7 with step 1), as well as defined for two norms (L_1, L_2) four parameters (U_x, U_y, p, ρ). Analyzing this visual image (Fig. 12), one can notice that the blue color (rCF solver) prevails on the surface over red (pCF solver), so the expert can choose this alternative (rCF slover).

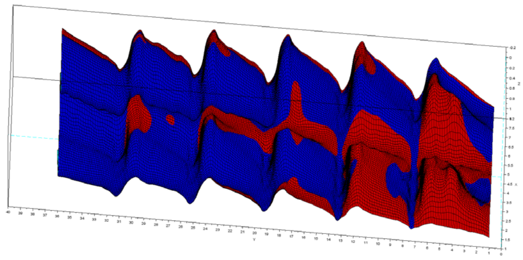

Fig. 12. Visual comparison of two solvers using surfaces in three-dimensional space

Fig. 13 shows the results of the visualization of alternatives using a probability distribution diagram. To build it, we used a partition of the values of the criteria into 10 intervals. In this diagram (Fig. 13), it can be noted that the blue color largely prevails in the columns [0.6; 0.7), [0.8; 0.9) and [0.9; 1.0], which correspond to the probability of falling into intervals with a higher value (rank). This means that for the normalized values of the rCF solver, the probability of obtaining a better solution is higher. Therefore, the choice of an expert, in this case, will most likely also be made in favor of this alternative (rCF solver).

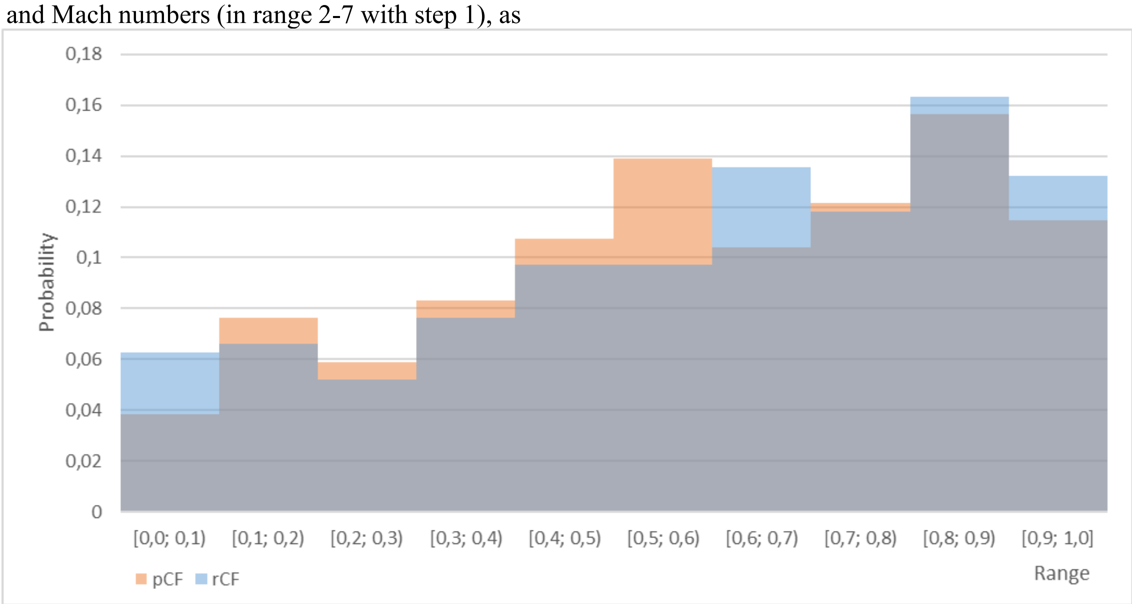

Fig. 13. Visual comparison of two solvers using a probability distribution diagram

Given the possibility of decomposing criteria into subsets, we consider the use of complex visual images based on petal families of diagrams. The decomposition will be carried out based on various parameters (U_x, U_y, p, ρ) and norms (L_1, L_2), i.e. for each alternative, we construct eight diagrams (Fig. 14).

Analyzing these visual images, it can be noted that for the first four rows (L_1 norm) due to a more uniform shape and subjectively somewhat larger area of images the second alternative (pCF solver) looks preferable, however, for the L_2 norm (5-8 rows) first alternative (rCF solver) is significantly preferable (due to a more uniform shape and a subjectively larger area of images). If we assume that these criteria are peer-to-peer, then it will be difficult for an expert to determine preference.

However, if these criteria can be ranked (for example, the criteria of the L_1 block are preferable to the criteria of the L_2 block, or vice versa), then it will be easier for an expert to make a choice because for this, it will suffice to compare either only the upper images or only the lower ones.

4. Conclusion

The analysis of visualization methods of alternatives for the pairwise comparison procedure showed that, depending on the properties of the source data and their criteria, various approaches can be both effective and not. Therefore, it is appropriate to attempt to use several different visualization methods and their combination in conjunction with a decomposition of the source data. In this case, it is possible on some methods to see that one alternative is better than another due to the subjective perception of the area of predominance, brightness, smoothness of forms, etc.

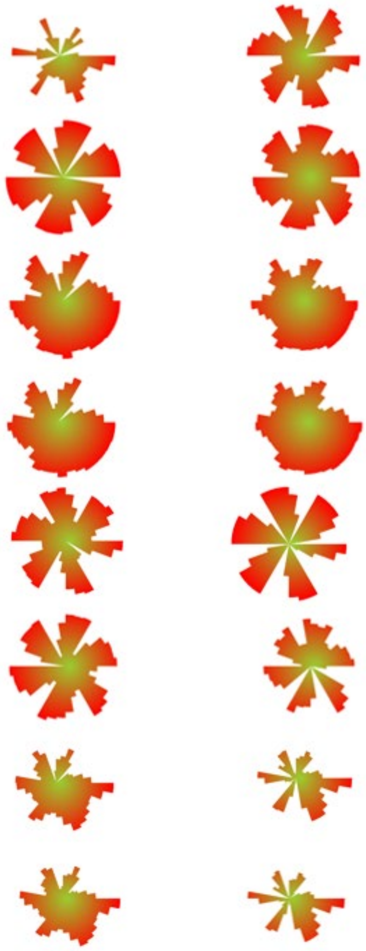

Fig. 14. Visual comparison of two solvers using a series of radar diagrams

The greatest effect in the pairwise comparison procedure can be achieved by visualization of groups of initial criteria combining with a ranking of decomposition parameters. Research in this direction can be quite promising. Those, we can thereby reduce the dimension of the initial data set, which will also allow us to apply traditional decision-making methods, and the comparison of criteria, in this case, can be based on the considered visualization methods or supplemented by them.

Acknowledgments

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References

[1] Castañón-Puga M., Sanchez M., Aguilar L., Rodríguez-Díaz A. Applied Decision-Making. Applications in Computer Sciences and Engineering. – 2019. DOI: 10.1007/978-3-030-17985-4.

[2] Alinezhad A., Khalili J. New Methods and Applications in Multiple Attribute Decision Making (MADM). – 2019. DOI: 10.1007/978-3-030-15009-9.

[3] David H.A. The Method of Paired Comparisons. (2nd edition, revised). – Oxford University Press, Berlin. – 1988.

[4] Saaty, T.L., 1980. Analytic Hierarchy Process: Planning, Priority Setting, and Resource Allocation. – New York: McGraw-Hill. – 1980.

[5] Averchenkov V.I., Miroshnikov V.V., Podvesovskiy A.G., Korostelyov D.A. (2014) Fuzzy and Hierarchical Models for Decision Support in Software Systems Implementations. Proceedings of the 11th Joint Conference, JCKBSE 2014 (eds. A.G. Kravets et. al.), Communications in Computer and Information Science, 466, Springer International Publishing, pp. 410-421. DOI: 10.1007/978-3-319-11854-3_35

[6] Practical application of the paired comparison method. – URL: https://usabilitylab.ru/blog/boevoe-primenenie-metoda-parnyix-sravnenij/.

[7] Figuera J., Greco S. and Ehrgott M. (Eds). Multiple Criteria Decision Analysis: State of the Art Surveys. – New York: Springer, 2005. – DOI: 10.1007/b100605.

[8] Zakharova A.A., Korostelyov D.A., Fedonin O.N. Visualization Algorithms for Multi-criteria Alternatives Filtering (2019). Scientific Visualization 11.4: 66 - 80, DOI: 10.26583/sv.11.4.06

[9] Bondarev A.E., Kuvshinnikov A.E. Analysis of the Accuracy of OpenFOAM Solvers for the Problem of Supersonic Flow Around a Cone // ICCS 2018, Lecture Notes in Computer Science (LNCS) 10862. – P. 221–230, 2018. – DOI:10.1007/978-3-319-93713-7_18.

[10] Bondarev A., Kuvshinnikov A. Comparative Estimation of QGDFoam Solver Accuracy for Inviscid Flow Around a Cone // IEEE The Proceedings of the 2018 Ivannikov ISPRAS Open Conference (ISPRAS-2018). – P. 82-87, 2018. – DOI: 10.1109/ISPRAS.2018.00019.

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